

一个与耦合 Harry-Dym 型方程相关的谱问题及其可积性

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摘要:通过二阶谱问题的相容性条件得到与其相关的非线性发展方程。利用位势函数与特征函数之间的联系得到了 Bargmann 系统, 并将发展方程族 Lax 对非线性化。建立合适的 Jacobi-Ostrogradsky 坐标, 得到一个有限维 Hamilton 正则系统。最后证明了其完全可积性并得到发展方程族的对合表示。

关键词:谱问题; Lax 对; Bargmann 系统; 可积系统

中图分类号:O175.9 **文献标志码:**A **文章编号:**2095-0373(2018)02-0106-05

0 引言

可积系统已经成为当代非线性科学的重要研究方向。Lax 对非线性化方法将无穷维可积系统约束为有限维可积系统, 并被广泛用于求解非线性发展方程^[1]。首先研究一个新的二阶特征值问题并得到其相对应的发展方程族, 其次利用 Lax 对非线性化方法获得有限维 Hamilton 系统, 并证明此系统在 Liouville 意义下的完全可积, 最后求解出相应的发展方程族在有限维空间上解的对合表示。

1 谱问题与发展方程族的 Lax 表示

讨论二阶线性谱问题

$$L\varphi = \varphi_{xx} + \lambda^3 u\varphi + \lambda^2 v\varphi + \lambda w\varphi = \alpha\varphi_x \quad (1)$$

式中, $u=u(x,t)$, $v=v(x,t)$, $w=w(x,t)$ 为位势函数; α 为常数; 特征参数 $\lambda \in \mathbb{R}$; $\varphi=\varphi(x,t)$ 为特征函数。

在基础空间 $\Omega=(-\infty, +\infty)$ 上讨论谱问题(1), 假设位势函数 u, v, w 及其关于 x 的各阶导数为无穷远速降函数。

命题 1 下列二阶谱问题构成完整谱系^[2]

$$\begin{cases} L\varphi = \varphi_{xx} + \lambda^3 u\varphi + \lambda^2 v\varphi + \lambda w\varphi = \alpha\varphi_x \\ \bar{L}\psi = \psi_{xx} + \lambda^3 u\psi + \lambda^2 v\psi + \lambda w\psi = -\alpha\psi_x \end{cases} \quad (2)$$

设谱问题(1)的辅谱问题为 $\varphi_{t_m} = w_m\varphi$, 其中

$$w_m = \sum_{j=0}^m \left(-\frac{1}{2}b_{jx} - \frac{1}{2}ab_j + b_j\partial \right) \lambda^{m-j+1} \quad (3)$$

令 $\mathbf{g}_j = (b_j, b_{j+1}, b_{j+2})^T$, $j=-1, 0, 1, \dots, m$, 则由相容性条件 $\varphi_{xxt} = \varphi_{txx}$, 得出如下递推关系: $\mathbf{K}\mathbf{g}_j = \mathbf{J}\mathbf{g}_{j+1}$, $j=-1, 0, 1, 2, \dots$, 其中

$$\mathbf{J} = \begin{pmatrix} 0 & \frac{1}{2}(\partial^3 - \alpha^2\partial) & 0 \\ \frac{1}{2}(\partial^3 - \alpha^2\partial) & \partial w + w\partial & 0 \\ 0 & 0 & -\partial u - u\partial \end{pmatrix}, \mathbf{K} = \begin{pmatrix} 0 & 0 & \frac{1}{2}(\partial^3 - \alpha^2\partial) \\ 0 & \frac{1}{2}(\partial^3 - \alpha^2\partial) & \partial w + w\partial \\ \frac{1}{2}(\partial^3 - \alpha^2\partial) & \partial w + w\partial & \partial v + v\partial \end{pmatrix},$$

收稿日期: 2016-10-27 责任编辑: 车轩玉 DOI: 10.13319/j.cnki.sjztdxxb.2018.02.20

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杨淑婷, 刘亚峰. 一个与耦合 Harry-Dym 型方程相关的谱问题及其可积性[J]. 石家庄铁道大学学报: 自然科学版, 2018, 31(2): 106-110.

取 $\mathbf{g}_{-1} = (b_{-1}, b_0, b_1)^T = (0, 0, u^{-\frac{1}{2}})^T$, 则有 $\mathbf{J}\mathbf{g}_{-1} = 0$, 并由递推关系得 $\{g_j, j = -1, 0, 1, \dots, m\}$, 特别 $\mathbf{g}_0 = (0, u^{-\frac{1}{2}}, -\frac{1}{2}vu^{-\frac{3}{2}})^T$, $\mathbf{g}_1 = (u^{-\frac{1}{2}}, -\frac{1}{2}vu^{-\frac{3}{2}}, -\frac{1}{2}wu^{-\frac{3}{2}} + \frac{3}{8}v^2u^{-\frac{5}{2}})^T$ 。

定理 1 在等谱条件下, 非线性发展方程族为

$$(u \ v \ w)_{t_m}^T = \mathbf{K}\mathbf{g}_m = \mathbf{J}\mathbf{g}_{m+1}, m = -1, 0, 1, \dots \quad (4)$$

对应的 Lax 对为

$$\begin{cases} L\varphi = \alpha\varphi_x \\ \varphi_{t_m} = \omega_m\varphi \end{cases} \quad m = -1, 0, 1, \dots \quad (5)$$

特别, 当 $m=0$ 时的发展方程为
$$\begin{cases} u_{t_0} = \frac{1}{2} \left[\left(\frac{1}{\sqrt{u}} \right)_{xxx} - \alpha^2 \left(\frac{1}{\sqrt{u}} \right)_x \right] \\ v_{t_0} = w_x \left(\frac{1}{\sqrt{u}} \right) + 2w \left(\frac{1}{\sqrt{u}} \right)_x \\ w_{t_0} = v_x \left(\frac{1}{\sqrt{u}} \right) + 2v \left(\frac{1}{\sqrt{u}} \right)_x \end{cases}, \text{这是一个耦合 Harry-Dym 型发展方程}^{[3]}。$$

2 Bargmann 系统与 Hamilton 正则系统

若 φ, ψ 是谱系 (2) 的特征值所对应的特征函数, 则 $\text{grad}\lambda = \begin{pmatrix} \partial\lambda/\partial w \\ \partial\lambda/\partial v \\ \partial\lambda/\partial u \end{pmatrix} =$

$$\left(-\int_{\Omega} (3\lambda^3 u + 2\lambda^2 v + w) \varphi \psi dx \right)^{-1} \begin{pmatrix} \lambda \varphi \psi \\ \lambda^2 \varphi \psi \\ \lambda^3 \varphi \psi \end{pmatrix}, \text{并有 } \mathbf{K} \text{ grad}\lambda = \lambda \mathbf{J} \text{ grad}\lambda。$$

设谱系 (2) 的 N 个不同的特征值 $\lambda_1 < \lambda_2 < \dots < \lambda_N$, Φ_j, Ψ_j 为对应于 $\lambda_j (j = 1, 2, \dots, N)$ 的特征函数, 令 $\Lambda = \text{diag}(\lambda_1, \lambda_2, \dots, \lambda_N)$, $\Phi = (\Phi_1, \Phi_2, \dots, \Phi_N)^T$, $\Psi = (\Psi_1, \Psi_2, \dots, \Psi_N)^T$ 。

作 Bargmann 约束, 令 $g_1 = (\langle \Lambda \Phi, \Psi \rangle, \langle \Lambda^2 \Phi, \Psi \rangle, \langle \Lambda^3 \Phi, \Psi \rangle)$ 得

$$u = \langle \Lambda \Phi, \Psi \rangle^{-2}, v = -2 \langle \Lambda^2 \Phi, \Psi \rangle \langle \Lambda \Phi, \Psi \rangle^{-3}, w = 3 \langle \Lambda^2 \Phi, \Psi \rangle^2 \langle \Lambda \Phi, \Psi \rangle^{-4} - 2 \langle \Lambda^3 \Phi, \Psi \rangle \langle \Lambda \Phi, \Psi \rangle^{-3}。$$

由此谱系 (2) 等价如下的 Bargmann 系统

$$\begin{cases} \Phi_{xx} + \Lambda^3 \langle \Lambda \Phi, \Psi \rangle^{-2} \Phi - 2\Lambda^2 \langle \Lambda^2 \Phi, \Psi \rangle \langle \Lambda \Phi, \Psi \rangle^{-3} \Phi + 3\Lambda \langle \Lambda^2 \Phi, \Psi \rangle^2 \langle \Lambda \Phi, \Psi \rangle^{-4} \Phi - 2\Lambda \langle \Lambda^3 \Phi, \Psi \rangle \langle \Lambda \Phi, \Psi \rangle^{-3} \Phi = \alpha \Phi_x, \\ \Psi_{xx} + \Lambda^3 \langle \Lambda \Phi, \Psi \rangle^{-2} \Psi - 2\Lambda^2 \langle \Lambda^2 \Phi, \Psi \rangle \langle \Lambda \Phi, \Psi \rangle^{-3} \Psi + 3\Lambda \langle \Lambda^2 \Phi, \Psi \rangle^2 \langle \Lambda \Phi, \Psi \rangle^{-4} \Psi - 2\Lambda \langle \Lambda^3 \Phi, \Psi \rangle \langle \Lambda \Phi, \Psi \rangle^{-3} \Psi = -\alpha \Psi_x, \end{cases} \quad (6)$$

定义 Lagrange 函数 $\hat{I} = \int_{\Omega} I dx^{[2]}$, 其中, $I = -\langle \Phi_x, \Psi_x \rangle - \langle \Lambda^2 \Phi, \Psi \rangle^2 \langle \Lambda \Phi, \Psi \rangle^{-3} + \langle \Lambda \Phi, \Psi \rangle^{-2} \langle \Lambda^3 \Phi, \Psi \rangle$,

$$\Psi \rangle - \frac{\alpha}{2} \langle \Phi_x, \Psi \rangle + \frac{\alpha}{2} \langle \Phi, \Psi_x \rangle。$$

命题 2 Bargmann 系统 (6) 等价于 Euler-Lagrange 方程: $\frac{\delta \hat{I}}{\delta \Phi} = 0, \frac{\delta \hat{I}}{\delta \Psi} = 0$ 。

设 u_1, u_2 为广义坐标, v_1, v_2 为广义动量, Hamilton 函数 $h = \sum_{j=1}^2 \langle u_{jx}, v_j \rangle - I$ 。

若取 $u_1 = \Phi, u_2 = \Psi$, 可直接计算得^[4] $v_1 = -\Psi_x - \frac{1}{2}\alpha\Psi, v_2 = -\Phi_x + \frac{1}{2}\alpha\Phi$ 。

构造 Jacobi-Ostrogradsky 坐标如下

$$y_1 = \Phi, y_2 = \Phi_x - \frac{1}{2}\alpha\Phi, z_1 = -\Psi_x - \frac{1}{2}\alpha\Psi, z_2 = \Psi \quad (7)$$

在此坐标系下, Bargmann 约束化为

$$\begin{cases} u = \langle \Lambda y_1, z_2 \rangle^{-2} \\ v = -2 \langle \Lambda^2 y_1, z_2 \rangle \langle \Lambda y_1, z_2 \rangle^{-3} \\ w = 3 \langle \Lambda^2 y_1, z_2 \rangle^2 \langle \Lambda y_1, z_2 \rangle^{-4} - 2 \langle \Lambda^3 y_1, z_2 \rangle \langle \Lambda y_1, z_2 \rangle^{-3} \end{cases} \quad (8)$$

由此 Bargmann 系统(6)等价于如下 Hamilton 正则系统

$$y_{jx} = \frac{\partial h}{\partial z_j}, \quad z_{jx} = -\frac{\partial h}{\partial y_j} \quad (9)$$

此时

$$h = \langle y_2, z_1 \rangle + \frac{\alpha}{2} \langle y_1, z_1 \rangle + \frac{\alpha}{2} \langle y_2, z_2 \rangle + \frac{\alpha^2}{4} \langle y_1, z_2 \rangle + \langle \Lambda^2 y_1, z_2 \rangle^2 \langle \Lambda y_1, z_2 \rangle^{-3} - \langle \Lambda y_1, z_2 \rangle^{-2} \langle \Lambda^3 y_1, z_2 \rangle \quad (10)$$

3 Hamilton 系统的完全可积性

在 Jacobi-Ostrogradsky 坐标(7)和 Bargmann 约束(8)下,应用非线性化方法^[2-3],发展方程族的 Lax 对(5)可表示为矩阵形式

$$\begin{cases} Y_x = MY, Z_x = -M^T Z; \\ Y_{t_m} = V_m Y, Z_{t_m} = -V_m^T Z; \quad m=0,1,\dots \end{cases} \quad (11)$$

其中

$$\begin{cases} Y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} \Phi \\ \Phi_x - \frac{1}{2}\alpha\Phi \end{pmatrix}, Z = \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \begin{pmatrix} -\Psi_x - \frac{1}{2}\alpha\Psi \\ \Psi \end{pmatrix} \\ M = \begin{pmatrix} \frac{1}{2}\alpha E & E \\ \frac{1}{4}\alpha^2 E - \Lambda^3 u - \Lambda^2 v - \Lambda w & \frac{1}{2}\alpha E \end{pmatrix} \end{cases} \quad (12)$$

$$E = E_{N \times N} = \text{diag}(1, 1, \dots, 1); V_m = (\omega_{ij}^m)_{2 \times 2} \quad i, j = 1, 2; m = 0, 1, 2, \dots$$

$$\omega_{11}^m = \sum_{j=0}^m -\frac{1}{2} b_{jx} \Lambda^{m-j+1} = \sum_{j=1}^m \left(\frac{1}{2} \langle \Lambda^j y_1, z_1 \rangle - \frac{1}{2} \langle \Lambda^j y_2, z_2 \rangle \right) \Lambda^{m-j+1};$$

$$\omega_{12}^m = \sum_{j=0}^m b_j \Lambda^{m-j+1} = \sum_{j=1}^m \langle \Lambda^j y_1, z_2 \rangle \Lambda^{m-j+1};$$

$$\omega_{21}^m = \sum_{j=0}^m \left(-\frac{1}{2} b_{jxx} + \frac{1}{4} \alpha^2 b_j - b_j \Lambda^3 u - b_j \Lambda^2 v - b_j \Lambda w \right) \Lambda^{m-j+1} =$$

$$\sum_{j=1}^m \langle \Lambda^j y_2, z_1 \rangle \Lambda^{m-j+1} + \sum_{j=1}^m \left(\langle \Lambda y_1, z_2 \rangle^{-2} \langle \Lambda^{j+3} y_1, z_2 \rangle - \Lambda^3 \langle \Lambda y_1, z_2 \rangle^{-2} \langle \Lambda^j y_1, z_2 \rangle^{-2} \langle \Lambda^j y_1, z_1 \rangle \right) \Lambda^{m-j+1} +$$

$$\sum_{j=1}^m \left(-2 \langle \Lambda^2 y_1, z_2 \rangle \langle \Lambda y_1, z_2 \rangle^{-3} \langle \Lambda^{j+2} y_1, z_2 \rangle + 2 \Lambda^2 \langle \Lambda^2 y_1, z_2 \rangle \langle \Lambda y_1, z_2 \rangle^{-3} \langle \Lambda^j y_1, z_2 \rangle \right) \Lambda^{m-j+1} +$$

$$\sum_{j=1}^m \left[3 \langle \Lambda^2 y_1, z_2 \rangle^2 \langle \Lambda y_1, z_2 \rangle^{-4} \langle \Lambda^{j+1} y_1, z_2 \rangle - 3 \Lambda \langle \Lambda^2 y_1, z_2 \rangle^2 \langle \Lambda y_1, z_2 \rangle^{-4} \langle \Lambda^j y_1, z_2 \rangle \right] \Lambda^{m-j+1};$$

$$\omega_{22}^m = \sum_{j=0}^m \frac{1}{2} b_{jx} \Lambda^{m-j+1} = \sum_{j=1}^m \left(\frac{1}{2} \langle \Lambda^j y_2, z_2 \rangle - \frac{1}{2} \langle \Lambda^j y_1, z_1 \rangle \right) \Lambda^{m-j+1}.$$

定理 3 在 Bargmann 约束条件(8)下,发展方程族的 Lax 对(5)等价于如下 Hamilton 正则方程^[5]

$$\begin{cases} Y_x = \frac{\partial h}{\partial z}, \quad Z_x = -\frac{\partial h}{\partial Y}; \\ Y_{t_m} = \frac{\partial h_m}{\partial Z}, \quad Z_{t_m} = -\frac{\partial h_m}{\partial Y}; \quad m=0,1,2,\dots \end{cases} \quad (13)$$

其中 Hamilton 函数 h 为式(10), h_m 为

$$\begin{aligned} h_m = & \langle \Lambda y_1, z_2 \rangle^{-2} \langle \Lambda^2 y_1, z_2 \rangle \langle \Lambda^{m+2} y_1, z_2 \rangle + \langle \Lambda y_1, z_2 \rangle^{-2} \langle \Lambda^3 y_1, z_2 \rangle \langle \Lambda^{m+1} y_1, z_2 \rangle - \\ & \langle \Lambda y_1, z_2 \rangle^{-1} \langle \Lambda^{m+3} y_1, z_2 \rangle - \langle \Lambda y_1, z_2 \rangle^{-3} \langle \Lambda^2 y_1, z_2 \rangle^2 \langle \Lambda^{m+1} y_1, z_2 \rangle + \\ & \sum_{j=1}^m \frac{1}{4} (\langle \Lambda^j y_1, z_1 \rangle + \langle \Lambda^j y_2, z_2 \rangle) (\langle \Lambda^{m-j+1} y_1, z_1 \rangle + \langle \Lambda^{m-j+1} y_2, z_2 \rangle) - \end{aligned}$$

$$\sum_{j=1}^m \left| \begin{array}{cc} \langle \Lambda^{m-j+1} y_1, z_1 \rangle & \langle \Lambda^j y_1, z_2 \rangle \\ \langle \Lambda^{m-j+1} y_2, z_1 \rangle & \langle \Lambda^j y_2, z_2 \rangle \end{array} \right|.$$

令

$$\begin{aligned} E_k^{(1)} &= \langle \Lambda y_1, z_2 \rangle^{-2} \langle \Lambda^2 y_1, z_2 \rangle \lambda_k^2 y_{1k} z_{1k} + \langle \Lambda y_1, z_2 \rangle^{-2} \langle \Lambda^3 y_1, z_2 \rangle \lambda_k y_{1k} z_{1k} - \\ &\quad \langle \Lambda y_1, z_2 \rangle^{-1} \lambda_k^3 y_{1k} z_{1k} - \langle \Lambda y_1, z_2 \rangle^{-3} \langle \Lambda^2 y_1, z_2 \rangle^2 \lambda_k y_{1k} z_{1k} - \\ &\quad \sum_{l=1, l \neq k}^N \frac{1}{\lambda_k - \lambda_l} (y_{1k} y_{2l} - y_{2k} y_{1l}) (z_{1k} z_{2l} - z_{2k} z_{1l}), \\ E_k^{(2)} &= \sum_{l=1, l \neq k}^N \frac{1}{\lambda_k - \lambda_l} (y_{1k} z_{1k} + y_{2k} z_{2k}) (y_{1l} z_{1l} + y_{2l} z_{2l}). \end{aligned}$$

定理 4:

(1)

$$\{E_j^m, E_k^n\} = 0, \forall j, k = 1, 2, \dots, N; m, n = 1, 2; \quad (14)$$

(2) $\{dE_j^m, j = 1, 2, \dots, N; m = 1, 2\}$ 线性无关;

$$(3) \mathfrak{Y}_\mu = \sum_{j=1}^N \frac{1}{\mu - \lambda_j} (E_j^1 + E_j^2) = \sum_{m=0}^{\infty} \mu^{-m-1} h_m, h_m = \sum_{j=1}^N \lambda_j^m (E_j^1 + E_j^2), m = 0, 1, 2, \dots.$$

定理 5 Hamilton 系统(13) 在 Liouville 意义下是完全可积系, 并且 Hamilton 相流 $g_{h_m}^{t_m}, g_{h_n}^{t_n}$ 可交换。

证明: 由式(14) 直接计算得 $\{\mathfrak{Y}_\mu, \mathfrak{Y}_\lambda\} = 0, \{h_m, h_n\} = 0, m, n = 0, 1, 2, \dots$,

直接计算可知 $\{h, E_j^l\} = 0, l = 1, 2; j = 0, 1, 2, \dots, N$, 所以 $\{h, h_m\} = 0, m = 0, 1, 2, \dots$ 。

由 Arnold 定理, Hamilton 系统(13)是完全可积系, 且 Hamilton 相流可换。

定理 6 设 (y_1, y_2, z_1, z_2) 满足 Hamilton 系统(13), 则式(8)为非线性发展方程族(4)的对合解。

4 结论

对所给定的二阶谱问题, 由相容性条件得到了一个耦合 Harry-Dym 型发展方程族, 通过 Bargmann 约束条件, 最终获得一个新的 Liouville 意义下的完全可积 Hamilton 正则系统。

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A Spectral Problem Related to Coupled Harry-Dym Type Equations and Its Integrability

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Abstract: Based on the compatibility conditions of the second order spectral, The related nonlinear evolution is obtained. By means of the constraint condition between the potentials and the eigenvector, the Bargmann system is obtained, and Lax pairs of the evolution equation are nonlinearized. The reasona-

ble Jacobi-Ostrogradsky coordinate has been found, and a new finite-dimensional Hamilton cononical equation is obtained. Finally, the completely integrable system is proved and the involutive solutions of the evolution equation is given.

Key words: spectral problem; Lax pairs; Bargmann system; integrable system

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## Research on the Model and Algorithm of the Train Working Diagram of High Speed Railway

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**Abstract:** Railway train timetable is the comprehensive plan in railway transport, and also, is the representation of transport products, especially in high speed railway. The product property of the train timetable has been becoming stronger. Train scheduling problem has the characteristic of strong constraints and combinatorial optimization. The paper is focused on the research of the model and algorithm of railway train timetable. With the goal of minimizing the total traveled time of all the trains, a regulation optimization model on the passenger line is set up based on comprehensive consideration of train operating time interval, stopping time, tracking interval time, train departure and arrival time domains and other safety constraints. The optimization algorithm based on genetic algorithm is also designed. An instance based on the background of Beijing-Shanghai passenger special line is constructed, and the genetic algorithm to the train regulation problem is programmed in MyEclipse10 simulation environment. Finally, the example is solved with genetic algorithm, and the simulation results are analyzed. The results prove the feasibility of the model and algorithm.

**Key words:** high speed railway; train diagram; genetic algorithm; simulation platform