

关于线路坐标计算公式的推导

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摘要: 运用已给的控制点坐标、方位角和线路曲线要素, 推导出一种计算线路中桩、边桩以及与线路非正交交叉的相关点位坐标(如桩基、附属物等)的简易方法, 从而给线路测设带来方便。

关键词: 平面线形; 线路坐标; 方位角

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随着 GPS 技术、测量仪器及计算机技术的飞速发展, 坐标测设法逐渐占据线路测设方法中的主导地位, 从而要求对线路坐标计算作出更深层次的分析。为了适应线路测设的要求, 研究线路中桩、边桩以及与线路非正交交叉的相关点位坐标(如桩基、附属物等)是很有必要的。在此介绍一种计算线路坐标的简易方法。该方法能够解算基本型、S 型、卵型、复合型、C 型等复杂的组合线形, 以基本型线形为例。

1 平面线形三要素

线路形式尽管很多, 但都是由平面线形三要素^[1]即直线、圆曲线和回旋线(缓和曲线)组成。当线路两端点的曲率为 0 时, 线形为直线; 当线路两端点的曲率相等且不为 0 时, 线形为圆曲线; 当线路两端点的曲率不相等时, 线形为缓和曲线(完整或不完整, 从曲率大到曲率小方向或从曲率小到曲率大方向)。

在平面设计中, 往往给出控制点坐标、方位角和线路曲线要素, 需求得线路中桩、边桩以及与线路非正交交叉的相关点位坐标。如图 1 所示, JD 为交点, BP 为起点, EP 为终点, 已知 JD 坐标 (X_J, Y_J) , BP 到 JD 方位角 α_1 , JD 到 EP 方位角 α_2 , 第一缓和曲线长度 L_{s1} , 第二缓和曲线长度 L_{s2} , 第一切线长度 T_1 , 第二切线长度 T_2 , 圆曲线半径 R 。由此可得出, 左、右两侧曲线的内移值 $p_1 = L_{s1}^2 / (24R)$, $p_2 = L_{s2}^2 / (24R)$, 左、右两侧曲线的切线增值 $q_1 = L_{s1} / 2 - L_{s1}^3 / (240R^2)$, $q_2 = L_{s2} / 2 - L_{s2}^3 / (240R^2)$ 。

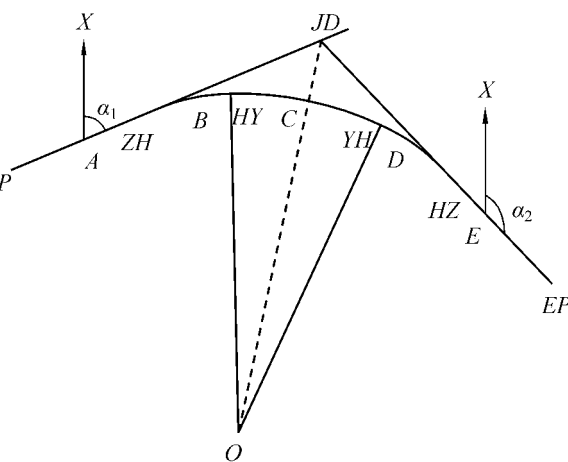


图 1 基本型平曲线

2 直线

2.1 交点前直线

设 (X_A, Y_A) 为直线上任意一点坐标, L 为该点与 JD 之间的距离, (X_{A1}, Y_{A1}) 为中桩 A 对应左侧边桩坐标, (X_{A2}, Y_{A2}) 为中桩 A 对应右侧边桩坐标, L_1 、 L_2 分别为中桩 A 至左、右侧边桩之间的距离, (X_{A3}, Y_{A3}) 为中桩 A 对应与线路交角 δ ($\delta \neq 90^\circ$) 的左侧相关点位坐标, (X_{A4}, Y_{A4}) 为中桩 A 对应与线路交角 δ ($\delta \neq 90^\circ$) 的右侧相关点位坐标, L_3 、 L_4 分别为中桩 A 至左、右侧相关点位之间的距离。

如图 2 所示, JD 到 BP 方位角 $\alpha_A = \angle XABP = \alpha_1 + 180^\circ + \xi 360^\circ$ 。则中桩 A 点坐标 $X_A = X_J +$

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$L\cos(\alpha_1 + 180^\circ + \xi 360^\circ)$, $Y_A = Y_J + L\sin(\alpha_1 + 180^\circ + \xi 360^\circ)$ 。其中,当方位角 $\alpha < 0$ 时, $\xi = 1$; 当方位角 $0 < \alpha < 360^\circ$ 时, $\xi = 0$; 当方位角 $\alpha > 360^\circ$ 时, $\xi = -1$, 其下 ξ 意义均相同。

如图3所示, A 到 A_1 方位角 $\alpha_{A1} = \angle XAA_1 = \alpha_1 - 90^\circ + \xi 360^\circ$, A 到 A_2 方位角 $\alpha_{A2} = \angle XAA_2 = \alpha_1 + 90^\circ + \xi 360^\circ$ 。则左侧边桩 A_1 点坐标 $X_{A1} = X_A + L_1\cos(\alpha_1 - 90^\circ + \xi 360^\circ)$, $Y_{A1} = Y_A + L_1\sin(\alpha_1 - 90^\circ + \xi 360^\circ)$; 右侧边桩 A_2 点坐标 $X_{A2} = X_A + L_2\cos(\alpha_1 + 90^\circ + \xi 360^\circ)$, $Y_{A2} = Y_A + L_2\sin(\alpha_1 + 90^\circ + \xi 360^\circ)$ 。

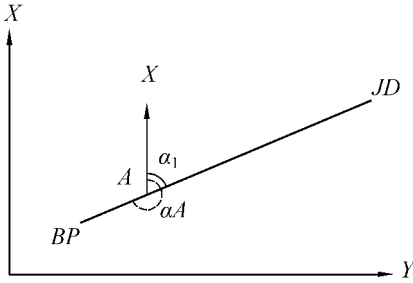


图2 交点前直线中桩

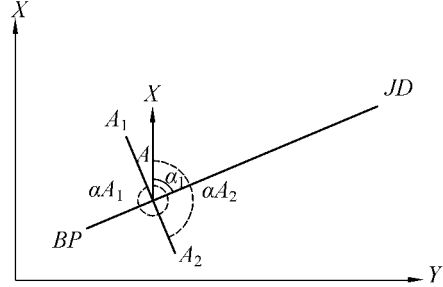


图3 交点前直线边桩

如图4所示, $\angle JDAA_4 = \delta$, A 到 A_3 方位角 $\alpha_{A3} = \angle XAA_3 = \alpha_1 + \delta - 180^\circ + \xi 360^\circ$, A 到 A_4 方位角 $\alpha_{A4} = \angle XAA_4 = \alpha_1 + \delta + \xi 360^\circ$ 。则左侧相关点 A_3 点坐标 $X_{A3} = X_A + L_3\cos(\alpha_1 + \delta - 180^\circ + \xi 360^\circ)$, $Y_{A3} = Y_A + L_3\sin(\alpha_1 + \delta - 180^\circ + \xi 360^\circ)$; 右侧相关点 A_4 点坐标 $X_{A4} = X_A + L_4\cos(\alpha_1 + \delta + \xi 360^\circ)$, $Y_{A4} = Y_A + L_4\sin(\alpha_1 + \delta + \xi 360^\circ)$ 。

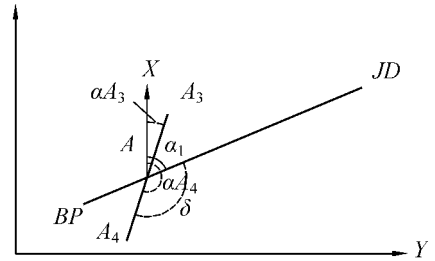


图4 交点前直线相关点位

2.2 交点后直线

设 (X_E, Y_E) 为直线上任意一点坐标, L' 为该点与 JD 之间的距离, (X_{E1}, Y_{E1}) 为中桩 E 对应左侧边桩坐标, (X_{E2}, Y_{E2}) 为中桩 E 对应右侧边桩坐标, L_1', L_2' 分别为中桩 E 至左、右侧边桩之间的距离, (X_{E3}, Y_{E3}) 为中桩 E 对应与线路交角 δ ($\delta \neq 90^\circ$) 的左侧相关点位坐标, (X_{E4}, Y_{E4}) 为中桩 E 对应与线路交角 δ ($\delta \neq 90^\circ$) 的右侧相关点位坐标, L_3', L_4' 分别为中桩 E 至左、右侧相关点位之间的距离。同交点前直线, 可得出

$$\begin{aligned} X_E &= X_J + L'\cos(\alpha_2 + \xi 360^\circ), Y_E = Y_J + L'\sin(\alpha_2 + \xi 360^\circ); \\ X_{E1} &= X_E + L_1'\cos(\alpha_2 - 90^\circ + \xi 360^\circ), Y_{E1} = Y_E + L_1'\sin(\alpha_2 - 90^\circ + \xi 360^\circ); \\ X_{E2} &= X_E + L_2'\cos(\alpha_2 + 90^\circ + \xi 360^\circ), Y_{E2} = Y_E + L_2'\sin(\alpha_2 + 90^\circ + \xi 360^\circ); \\ X_{E3} &= X_E + L_3'\cos(\alpha_2 + \delta - 180^\circ + \xi 360^\circ), Y_{E3} = Y_E + L_3'\sin(\alpha_2 + \delta - 180^\circ + \xi 360^\circ); \\ X_{E4} &= X_E + L_4'\cos(\alpha_2 + \delta + \xi 360^\circ), Y_{E4} = Y_E + L_4'\sin(\alpha_2 + \delta + \xi 360^\circ). \end{aligned}$$

3 缓和曲线

3.1 第一缓和曲线

设 (X_B, Y_B) 为第一缓和曲线上任意一点坐标, r 为该点与 ZH 点之间的距离, (X_{B1}, Y_{B1}) 为中桩 B 对应左侧边桩坐标, (X_{B2}, Y_{B2}) 为中桩 B 对应右侧边桩坐标, r_1, r_2 分别为中桩 B 至左、右侧边桩之间的距离, (X_{B3}, Y_{B3}) 为中桩 B 对应与线路交角 δ ($\delta \neq 90^\circ$) 的左侧相关点位坐标, (X_{B4}, Y_{B4}) 为中桩 B 对应与线路交角 δ ($\delta \neq 90^\circ$) 的右侧相关点位坐标, r_3, r_4 分别为中桩 B 至左、右侧相关点位之间的距离。

由交点前直线可知, ZH 点中桩坐标 $X_{ZH} = X_J + T_1\cos(\alpha_1 + 180^\circ + \xi 360^\circ)$, $Y_{ZH} = X_J + T_1\sin(\alpha_1 + 180^\circ + \xi 360^\circ)$ 。如图5所示, $BM \parallel BPJD$, β_1 为 BM 与 BN 之间的夹角, ZH 到 B 方位角 $\alpha_B = \angle XBN = \alpha_1 + \eta\beta_1 + \xi 360^\circ$ 。则中桩 B 点坐标 $X_B = X_{ZH} + r\cos(\alpha_1 + \eta\beta_1 + \xi 360^\circ)$, $Y_B = Y_{ZH} + r\sin(\alpha_1 + \eta\beta_1 + \xi 360^\circ)$ 。

其中, $\beta_1 = \arctan(y/x)$, $x = l - \frac{l^5}{40R^2L_{S1}^2} + \frac{l^9}{3456R^4L_{S1}^4} - \frac{l^{13}}{599040R^6L_{S1}^6} + \dots$, $y = \frac{l^3}{6RL_{S1}} - \frac{l^7}{336R^3L_{S1}^3} + \dots$

$\frac{l^{11}}{42\,240R^5L_{S1}^5} - \frac{l^{15}}{96\,768\,000R^7L_{S1}^7} + \dots, r = \sqrt{x^2 + y^2}, l$ 为第一缓和曲线上任意点至 ZH 点的曲线长, η 为转角符号, 右转角为“+”, 左转角为“-”。

如图 6 所示, BQ 为 B 点处切线, β_{x1} 为 BM 与 BQ 之间的夹角(切线角), B 到 B_1 方位角 $\alpha_{B1} = \angle XBB_1 = \alpha_1 + \eta\beta_{x1} - 90^\circ + \xi 360^\circ$, B 到 B_2 方位角 $\alpha_{B2} = \angle XBB_2 = \alpha_1 + \eta\beta_{x1} + 90^\circ + \xi 360^\circ$ 。则左侧边桩 B_1 点坐标 $X_{B1} = X_B + r_1 \cos(\alpha_1 + \eta\beta_{x1} - 90^\circ + \xi 360^\circ)$, $Y_{B1} = Y_B + r_1 \sin(\alpha_1 + \eta\beta_{x1} - 90^\circ + \xi 360^\circ)$; 右侧边桩 B_2 点坐标 $X_{B2} = X_B + r_2 \cos(\alpha_1 + \eta\beta_{x1} + 90^\circ + \xi 360^\circ)$, $Y_{B2} = Y_B + r_2 \sin(\alpha_1 + \eta\beta_{x1} + 90^\circ + \xi 360^\circ)$ 。其中, $\beta_{x1} = 90l^2/(\pi L_{S1}R)$, l 为第一缓和曲线上任意点至 ZH 点的曲线长, η 为转角符号, 右转角为“+”, 左转角为“-”。

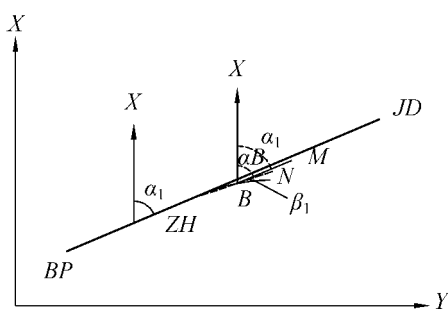


图 5 第一缓和曲线中桩

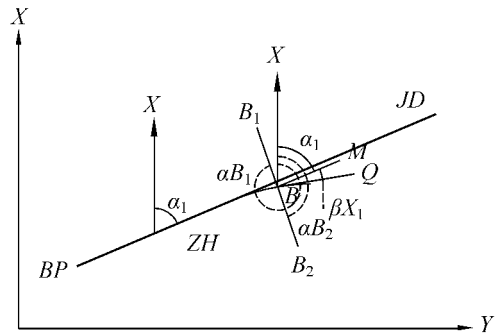


图 6 第一缓和曲线边桩

如图 7 所示, $\angle QBB_4 = \delta$, B 到 B_3 方位角 $\alpha_{B3} = \angle XBB_3 = \alpha_1 + \eta\beta_{x1} + \delta - 180^\circ + \xi 360^\circ$, B 到 B_4 方位角 $\alpha_{B4} = \angle XBB_4 = \alpha_1 + \eta\beta_{x1} + \delta + \xi 360^\circ$ 。则左侧相关点位 B_3 点坐标 $X_{B3} = X_B + r_3 \cos(\alpha_1 + \eta\beta_{x1} + \delta - 180^\circ + \xi 360^\circ)$, $Y_{B3} = Y_B + r_3 \sin(\alpha_1 + \eta\beta_{x1} + \delta - 180^\circ + \xi 360^\circ)$; 右侧相关点位 B_4 点坐标 $X_{B4} = X_B + r_4 \cos(\alpha_1 + \eta\beta_{x1} + \delta + \xi 360^\circ)$, $Y_{B4} = Y_B + r_4 \sin(\alpha_1 + \eta\beta_{x1} + \delta + \xi 360^\circ)$ 。

3.2 第二缓和曲线

设 (X_D, Y_D) 为第二缓和曲线上任意一点坐标, r' 为该点与 HZ 点之间的距离, (X_{D1}, Y_{D1}) 为中桩 D 对应左侧边桩坐标, (X_{D2}, Y_{D2}) 为中桩 D 对应右侧边桩坐标, r'_1, r'_2 分别为中桩 D 至左、右侧边桩之间的距离, (X_{D3}, Y_{D3}) 为中桩 D 对应与线路交角 δ ($\delta \neq 90^\circ$) 的左侧相关点位坐标, (X_{D4}, Y_{D4}) 为中桩 D 对应与线路交角 δ ($\delta \neq 90^\circ$) 的右侧相关点位坐标, r'_3, r'_4 分别为中桩 D 至左、右侧相关点位之间的距离。同第一缓和曲线, 可得出

$$X_{HZ} = X_J + T_2 \cos(\alpha_2 + \xi 360^\circ), Y_{HZ} = Y_J + T_2 \sin(\alpha_2 + \xi 360^\circ);$$

$$X_D = X_{HZ} + r' \cos(\alpha_2 - \eta\beta_2 + 180^\circ + \xi 360^\circ), Y_D = Y_{HZ} + r' \sin(\alpha_2 - \eta\beta_2 + 180^\circ + \xi 360^\circ);$$

$$X_{D1} = X_D + r'_1 \cos(\alpha_2 - \eta\beta_{x2} - 90^\circ + \xi 360^\circ), Y_{D1} = Y_D + r'_1 \sin(\alpha_2 - \eta\beta_{x2} - 90^\circ + \xi 360^\circ);$$

$$X_{D2} = X_D + r'_2 \cos(\alpha_2 - \eta\beta_{x2} + 90^\circ + \xi 360^\circ), Y_{D2} = Y_D + r'_2 \sin(\alpha_2 - \eta\beta_{x2} + 90^\circ + \xi 360^\circ);$$

$$X_{D3} = X_D + r'_3 \cos(\alpha_2 - \eta\beta_{x2} + \delta - 180^\circ + \xi 360^\circ), Y_{D3} = Y_D + r'_3 \sin(\alpha_2 - \eta\beta_{x2} + \delta - 180^\circ + \xi 360^\circ);$$

$$X_{D4} = X_D + r'_4 \cos(\alpha_2 - \eta\beta_{x2} + \delta + \xi 360^\circ), Y_{D4} = Y_D + r'_4 \sin(\alpha_2 - \eta\beta_{x2} + \delta + \xi 360^\circ)。$$

其中, $\beta_2 = \arctan(y'/x')$, $\beta_{x2} = 90l^2/(\pi L_{S2}R)$, $x' = l - \frac{l^5}{40R^2L_{S2}^2} + \frac{l^9}{3456R^4L_{S2}^4} - \frac{l^{13}}{599\,040R^6L_{S2}^6} + \dots$, $y' = \frac{l^3}{6RL_{S2}} - \frac{l^7}{336R^3L_{S2}^3} + \frac{l^{11}}{42\,240R^5L_{S2}^5} - \frac{l^{15}}{96\,768\,000R^7L_{S2}^7} + \dots$, $r' = \sqrt{x'^2 + y'^2}$, l 为第二缓和曲线上任意点至 HZ 点的曲线长, η 为转角符号, 右转角为“+”, 左转角为“-”。

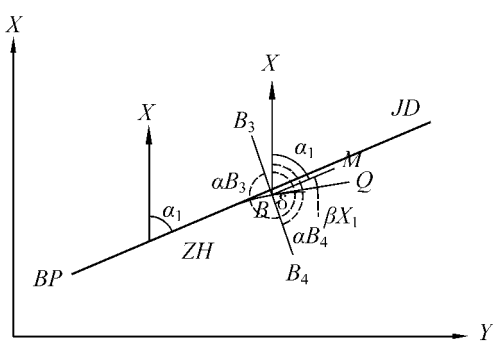


图 7 第一缓和曲线相关点位

4 圆曲线

设 (X_C, Y_C) 为圆曲线上任意一点坐标, (X_{C1}, Y_{C1}) 为中桩 C 对应左侧边桩坐标, (X_{C2}, Y_{C2}) 为中桩 C 对应右侧边桩坐标, R_1, R_2 分别为中桩 C 至左、右侧边桩之间的距离, (X_{C3}, Y_{C3}) 为中桩 C 对应与线路交角 δ ($\delta \neq 90^\circ$) 的左侧相关点位坐标, (X_{C4}, Y_{C4}) 为中桩 C 对应与线路交角 δ ($\delta \neq 90^\circ$) 的右侧相关点位坐标, R_3, R_4 分别为中桩 C 至左、右侧相关点位之间的距离。如图 8 所示, $CM' \parallel BPJD$, CN' 为 HY 点处切线的平行线, CC' 为 C 点处切线, $OG \perp BPJD$, $JDF \parallel OG$, β 为第一缓和曲线终点 HY 点处的切线角, θ 为 HY 点到 C 点的圆心角, 由此可得, $\gamma = \angle JDOG = \arctan \frac{T_1 - q_1}{R + p_1}$, $E = \frac{R + p_1}{\cos \gamma} - R$ 。 JD 到圆心 O 方位角 $\alpha_0 = \angle XJDO = \alpha_1 + \eta\gamma + (180^\circ - \eta 90^\circ) + \xi 360^\circ$, 圆心 O 到 C 方位角 $\alpha_c = \angle XCC_1 = \alpha_1 + \eta\beta + \eta\theta - \eta 90^\circ + \xi 360^\circ$, C 到 C_1 方位角 $\alpha_{c1} = \alpha_c = \alpha_1 + \eta\beta + \eta\theta - 90^\circ + \xi 360^\circ$, C 到 C_2 方位角 $\alpha_{c2} = \angle XCC_2 = \alpha_1 + \eta\beta + \eta\theta + 90^\circ + \xi 360^\circ$ 。则 $X_o = X_J + (R + E) \cos(\alpha_1 + \eta\gamma + (180^\circ - \eta 90^\circ) + \xi 360^\circ)$, $Y_o = Y_J + (R + E) \cdot \sin(\alpha_1 + \eta\gamma + (180^\circ - \eta 90^\circ) + \xi 360^\circ)$; $X_C = X_o + R \cos(\alpha_1 + \eta\beta + \eta\theta - \eta 90^\circ + \xi 360^\circ)$, $Y_C = Y_o + R \sin(\alpha_1 + \eta\beta + \eta\theta - \eta 90^\circ + \xi 360^\circ)$; $X_{C1} = X_C + R_1 \cos(\alpha_1 + \eta\beta + \eta\theta - 90^\circ + \xi 360^\circ)$, $Y_{C1} = Y_C + R_1 \sin(\alpha_1 + \eta\beta + \eta\theta - 90^\circ + \xi 360^\circ)$; $X_{C2} = X_C + R_2 \cos(\alpha_1 + \eta\beta + \eta\theta + 90^\circ + \xi 360^\circ)$, $Y_{C2} = Y_C + R_2 \sin(\alpha_1 + \eta\beta + \eta\theta + 90^\circ + \xi 360^\circ)$ 。

其中, $\beta = 90L_{s1}/(\pi R)$, $\theta = 180l/(\pi R)$, l 为圆曲线上任意点至 HY 点的曲线长, η 为转角符号, 右转角为“+”, 左转角为“-”。

如图 9 所示, $\angle C'CC_4 = \delta$, C 到 C_3 方位角 $\alpha_{c3} = \angle XCC_3 = \alpha_1 + \eta\beta + \eta\theta + \delta - 180^\circ + \xi 360^\circ$, C 到 C_4 方位角 $\alpha_{c4} = \angle XCC_4 = \alpha_1 + \eta\beta + \eta\theta + \delta + \xi 360^\circ$ 。则左侧相关点位 C_3 点坐标 $X_{C3} = X_C + R_3 \cos(\alpha_1 + \eta\beta + \eta\theta + \delta - 180^\circ + \xi 360^\circ)$, $Y_{C3} = Y_C + R_3 \sin(\alpha_1 + \eta\beta + \eta\theta + \delta - 180^\circ + \xi 360^\circ)$; 右侧相关点位 C_4 点坐标 $X_{C4} = X_C + R_4 \cos(\alpha_1 + \eta\beta + \eta\theta + \delta + \xi 360^\circ)$, $Y_{C4} = Y_C + R_4 \sin(\alpha_1 + \eta\beta + \eta\theta + \delta + \xi 360^\circ)$ 。

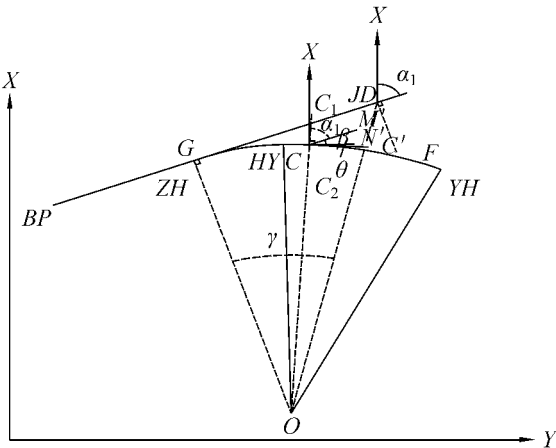


图 8 圆曲线中桩及其边桩

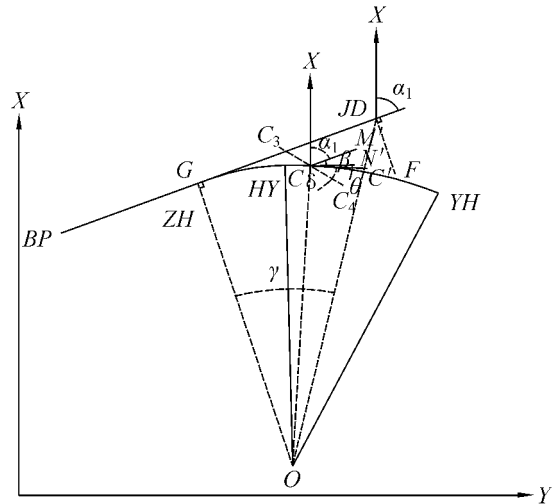


图 9 圆曲线相关点位

5 结语

通过理论推导得出线路中桩、边桩以及与线路非正交交叉的相关点位坐标, 其关键是先计算出待求点相对于特征点方位角, 进而可计算出各点坐标。此计算公式虽然繁冗, 但是形式简单, 可编程计算, 并且可以解算基本型、S 型、卵型、凸型、复合型、C 型等诸多复杂线形, 具有很强的实用性。

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On Calculation Formula of Line Coordinates

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Abstract: This paper makes full use of the control point coordinates, azimuth and line curves of elements, to derive a simple method to calculate the coordinates of central and edge stakes of the line and relevant coordinates of non-orthogonal cross-points (such as pile, adjunct, etc.) and, providing convenience for route survey and design.

Key words: planar alignment; line coordinates; azimuth

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## Analysis on Vertical Dynamic Response of Rigid-frame Bridge and Vehicle Interaction

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**Abstract:** Taking the typical three-span continuous rigid-framed bridge with each span of 36 m in Guangzhou as research object, a spatial model about coupled vibration between the train and the bridge is built by means of program. Diversification rules of impact factor and vertical acceleration are summarized by analyzing a large quantity of structure parameters and vehicle moving speed. The results indicate that impact factor in the middle of side span is larger than any other locations and by the increasing of beam rigidity and pier rigidity the impact factor is increased. Vehicle moving speed is one of the most important influence factors to the responses of bridge and vehicle. Impact factor increases distinctly when loading frequency approaches to the first vertical natural frequency. Vehicle vertical acceleration is reduced by the increasing of beam vertical rigidity and is increased by the elevating of vehicle velocity.

**Key words:** railway transportation; rigid-framed bridge; coupling vibration; impact factor

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