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磁电弹性材料断裂力学研究现状

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摘要:磁电弹性材料断裂问题已成为力学研究工作者的研究热点。全面回顾了近年来磁电弹性材料断裂力学的研究现状,主要讨论了:反平面裂纹问题、平面裂纹问题、硬币型裂纹问题以及有限元、边界元和无网格法在磁电弹性材料断裂问题中的应用。

关键词:磁电弹性材料;断裂力学;无网格法

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0 引言

磁电弹性材料是一种新型的智能材料,它能实现磁能、电能、机械能彼此之间的转换,即当由外力引起变形时这种材料可以产生电场和磁场,而在材料上施加电场或磁场则会引起材料的变形。

早在1894年,Curie在研究晶体对称性后,指出非对称分子晶体在外磁场作用下有可能定向极化。后来,Landau和Lifshitz[1]基于热力学和对称性考虑,认为磁有序晶体中可能存在线性的磁电效应。Dzyaloshinsky[2]根据理论分析指出，在反铁磁物质 Cr_2O_3 中存在磁电效应。随后,Astrov[3]测量了 Cr_2O_3 在磁场作用下的感生电场,Rado和Folen[4]探测到 Cr_2O_3 由电场极化而感生的磁场。

磁电弹性材料的多场耦合特性,使其在传感技术、信息技术、新兴的智能材料系统与结构以及微机电系统等高新技术领域显示出十分诱人的应用前景。

磁电弹性材料的固有弱点是力学性能上的脆性,在工作状态下由机械、电或磁载荷引起的应力集中会导致裂纹的产生和扩展,最终将造成部件的失效。事实上,磁电弹性材料本身也往往预先存在微裂纹、分层、空洞或夹杂等缺陷,它们也是引起部件失效的重要根源。因此,从力电磁耦合的观点出发,用力学理论对磁电弹性材料的断裂过程作理论分析和精确的定量描述,对研制和设计高品质的磁电弹性装置,提高磁电弹性器件的工作性能并对其可靠工作寿命进行预估,以及正确认识和理解磁电弹性材料的失效行为都具有重要的实际应用价值和学术意义。

1 磁电弹性材料断裂力学问题研究现状

对于磁电弹性材料断裂问题,主要是研究裂纹尖端附近磁电弹性场的性质,并给出断裂特征参数,如场强度因子(包括应力、电位移和磁感应强度因子)、能量密度因子和能量释放率等,以便对裂纹的扩展规律进行研究。

磁电弹性材料断裂参数获得的主要方法有Stroh方法、Lekhnitskii复势方法、积分变换法和分布位错法等等。

关于磁电弹性材料断裂问题的微细观分析可见以下文献:Li[5]研究了磁电弹性材料多夹杂和非均匀体问题。Wang和Shen[6]研究了磁电弹性复合材料任意夹杂问题。Fang等[7]和Zheng等[8]研究了磁电弹

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性螺位错和界面刚性线或界面裂纹相互作用问题。Shen[9]研究了磁电弹性螺位错与圆形层状夹杂相互作用问题。

关于磁电弹性材料断裂问题宏观分析分以下四方面分别叙述。

1.1 反平面裂纹问题

Spyropoulos 等[10]研究了磁电弹性材料反平面裂纹问题,电磁极化方向平行于裂纹面,结果发现:当 $BaTiO_3$-$CoFe_2O_4$ 复合材料体积分数为 50% 时裂纹易扩展,而体积分数增大或减小都会抑制裂纹扩展。Gao 等人[11]将 Stroh 方法推广到反平面变形问题中,研究了含椭圆孔洞的磁电弹性介质,在远场均布平面电磁载荷及反平面机械载荷作用下的 III 型断裂力学问题,分别给出了可导通和不可导通两种磁电边界条件下 III 型裂纹问题的封闭解。Wang 和 Mai[12]得到了磁电弹性材料反平面裂纹问题的封闭解,导出了能量释放率,并指出电或磁载荷会产生负的能量释放率。Wang 等[13-14]研究了磁电弹性材料反平面裂纹问题,分别考虑了四种理想裂纹面磁电边界条件,分析了单一裂纹、两个共线裂纹、周期排列平行裂纹、两个周期排列平行裂纹问题及反平面界面裂纹问题[15]。Wang 和 Mai[16]研究了磁电弹性条反平面裂纹问题,考虑了两种裂纹面边界条件和多种裂纹情况。Zhou 及其合作者[17-23]采用 Schmit 方法分别研究了磁电弹材料及功能梯度磁电弹性材料中的多种反平面裂纹问题。Soh 和 Liu[24]研究了圆弧形反平面界面裂纹问题,该裂纹模型可用来模拟不同材料界面脱粘问题。Chue 和 Liu[25]研究了含有界面裂纹的磁电弹性楔体反平面断裂问题,并指出,由于问题的对称性,应力、电位移和磁感应是不耦合的,并与材料参数无关,但是应变、电场和磁场仍然相互耦合。Hu 和 Li[26]研究了在纵向剪切荷载作用下含有限 Griffith 裂纹的无限长磁电弹条形介质,利用积分变换技术得到了反平面裂纹问题的封闭解,给出了八种可能加载情况下的场强度因子和能量释放率的显式表达式。Hu 等[27]研究了含裂纹磁电弹性层分别粘接两个弹性半空间的问题。Hao 和 Liu[28]研究了磁电弹性材料反平面界面半无限裂纹问题。Li 和 Kardomateas[29]研究了磁电弹性双材料界面 III 型裂纹问题。Liang[30]研究了磁电弹性材料两个共线界面裂纹的反平面断裂问题。Li 和 Lee[31-32]研究了功能梯度磁电弹性层界面裂纹和裂纹穿过两个相互粘接的功能梯度磁电弹性层的反平面裂纹问题。Su 和 Feng[33]研究了磁电弹性矩形板界面反平面裂纹问题。Guo 等[34]研究了含有裂纹的功能梯度磁电弹性层粘接两个不同功能梯度层的反平面断裂问题。Li 等[35]研究了含裂纹两个磁电弹性层粘结的问题,裂纹垂直并终止于界面,结果表明:双材料与均匀材料两种情况裂纹的应力场具有不同的奇异性。Ma 等[36]研究了含裂纹磁电弹性层和弹性层粘结在一起的问题,其中裂纹垂直于材料层面,发现:磁载荷会加速裂纹扩展,而电载荷与之相反。Singh 等[37]给出了磁电弹性材料两个共线反平面裂纹问题的封闭解。

对于动态断裂问题,Feng 等人研究了无限大磁电弹性材料反平面单裂纹[38]、矩形板单一裂纹[39]、有限厚度条多裂纹[40]及功能梯度磁电弹性材料反平面裂纹[41-42]问题。Li[43]和 Yong 和 Zhou[44]研究了极化方向与裂纹面平行时磁电弹性材料反平面裂纹问题,分别考虑了裂纹平行于材料边界和垂直于材料边界两种情况。Hu 分别研究了磁电弹性材料内部[45]和界面[46]及矩形体内[47]移动反平面裂纹问题。Zhou 等[48]利用 Schmidt 方法分析了两种不同压电压磁复合材料界面可导通单裂纹和共线裂纹对反平面简谐波的散射问题。Liang[49]研究了功能梯度磁电弹性材料两个平行对称反平面裂纹问题。Peng 和 Li[50]研究了磁电弹性半空间与一功能梯度层界面裂纹的动态断裂问题。给出了应力与应变强度因子、能量释放率和能量密度因子,并指出这种情况下应变强度因子作为断裂准则最为有效。Tupholme[51]研究了横观各向同性磁电弹性材料移动反平面裂纹问题。杜建科等[52]研究了横观各向同性磁电弹性介质中磁电不导通型界面裂纹对 SH 波的散射,他们将混合边值问题转换为柯西奇异积分方程组,然后数值求解得到了动应力强度因子随入射波频率的变化规律。应用扰动理论和波函数展开的方法,Feng 等[53-54]研究了压电-压磁复合材料中圆弧型界面裂纹对 SH 波的散射问题,通过数值算例讨论了入射角和裂纹张开角对动应力强度因子的影响。Zhou 等[55-56]研究了两个平行裂纹受反平面剪切波作用的问题。Liang[57]研究了功能梯度反平面剪切波的散射问题。

1.2 平面裂纹问题

Sih 和 Chen[58]从能量密度的观点分析了含裂纹磁电弹性材料的变形模式。Sih 等[59]研究了磁电弹性材料Ⅰ、Ⅱ型裂纹问题,讨论了不同电磁极化角度和 $BaTiO_3$-$CoFe_2O_4$ 复合材料体积分数对能量密度因子的影响。Song 和 Sih[60]研究了含裂纹的磁电弹性介质平面变形问题,导出了应力强度因子和能量密度因子解,并分别对Ⅰ型、Ⅱ型和复合型裂纹扩展行为进行了分析,认为裂纹扩展的方向依赖于所加电磁场的大小和方向。Sih 和 Song[61]研究了含裂纹的磁电弹性材料平面问题,考虑了四种组合极化方向下 $BaTiO_3$-$CoFe_2O_4$ 复合材料体积分数对能量密度因子的影响。Sih 和 Yu[62]讨论了 $BaTiO_3$-$CoFe_2O_4$ 复合材料体积分数对磁电弹性材料Ⅰ、Ⅱ型裂纹能量密度因子的影响。

Gao 等[63-64]分别研究了在远场载荷作用下无限大二维各向异性磁电弹性介质中可导通单一裂纹和共线裂纹问题,采用广义 Eshelby-Stroh 方法,给出了问题的精确解,得出的结论是,裂纹尖端由于磁电弹性场耦合作用产生的奇异性只取决于外加机械荷载,与外加电场和磁场无关,应力强度因子与各向同性材料相同。Gao 等[65]采用可导通磁电边界条件给出了两相磁电弹性介质界面裂纹问题的解析解,并利用解析解分别研究了在远场均布载荷、广义线载荷和广义位错作用下的解,分析了裂纹尖端的振荡奇异性。Gao 和 Noda[66]研究了磁电弹性双材料界面裂纹在均匀热流作用下的断裂问题。

Wang 和 Mai[67]得到了含裂纹磁电弹性介质二维问题的解析解,给出了能量释放率表达式,结果表明:无机械载荷作用时,电磁载荷作用总产生负的能量释放率。Wang[68]讨论了磁电弹性材料平面裂纹磁电边界条件的适用性问题。Tian 和 Gabbert[69-74]分别研究了磁电弹性材料中的单一裂纹、多裂纹、缓慢扩展裂纹、半无限宏微观裂纹相互作用、分叉裂纹问题。Zhang 等[75]研究了磁电弹性材料两个平行Ⅰ型裂纹问题。Zhong 和 Li[76]研究了磁电弹性材料平面裂纹问题,考虑了裂纹内的电磁特性。Zhong 和 Li[77]给出了磁电弹性材料平面裂纹问题的 T 应力。Zhong[78]以裂纹张开位移(COD)强度因子作为断裂准则研究了磁电弹性层不导通裂纹问题。Zhou 等[79-82]采用 Schmit 方法分别研究了磁电弹材料及功能梯度磁电弹性材料中的Ⅰ型裂纹问题。Li 和 Kardomateas[83]研究了磁电弹性双材料界面Ⅰ、Ⅱ型裂纹问题。Zhao 和 Fan[84]提出了磁电弹性材料平面裂纹的电磁击穿模型,分析了非线性电磁场对裂纹的影响。Huang 和 Wang[85]分析了磁电弹性材料界面裂纹问题,并指出界面裂纹的存在将弱化复合材料的耦合性质。Fan 等[86]分析了二维磁电弹性材料界面裂纹的应力奇异性问题。

Zhong 等[87]研究了磁电弹性材料两个共线裂纹的动态断裂问题,考虑了裂纹内部的磁电性质。Feng 等[88]研究了两个不同磁电弹性层界面Ⅰ型裂纹的动态断裂问题,分别考虑了四种不同的磁电边界条件。

1.3 硬币型裂纹问题

Hou 和 Leung[89]得到了磁电弹性材料球形夹杂封闭解,并由此导出了硬币型裂纹的场强度因子解。Zhao 等[90]考虑裂纹磁电导通性得到了硬币型裂纹的闭合解,并评价了导通与不导通裂纹模型的适用性问题。Feng 等[91]研究了在给定广义应力或位移时磁电弹性层硬币型裂纹的动态断裂行为。Wang 等[92]提出了一个自适应断裂模型以确定硬币型裂纹的形状及裂纹内的磁电场,考虑了裂纹内部空气或真空的磁电性能与裂纹变形的耦合行为。Zhong 和 Li[93]考虑裂纹可导通电磁场情况,分析了磁电弹性材料硬币型裂纹问题。Tian 和 Rajapakse[94]导出了磁电弹性层硬币型裂纹的场强度因子。Niraula 和 Wang[95-96]研究了磁电弹性材料硬币型裂纹在温度载荷或热流作用下的断裂问题。

Li 等[97]研究了功能梯度磁电弹性层硬币型裂纹在机械载荷作用下的动态断裂问题。Feng 等[98]研究了均匀磁电弹性层硬币型裂纹动态断裂问题。

Zhao 等[99-100]采用超奇异边界积分方程方法研究了三维磁电弹性介质内任意平面裂纹问题。

1.4 断裂问题的数值求解

过去的几十年来,有限元及边界元法获得了很大发展,在断裂力学问题中得到了广泛的应用。在磁电弹性材料的断裂问题中也取得了一些成果,如应用有限元法[101-102]和边界元法[103-105]解决力电磁载荷作用下的静动态断裂问题。

近年来,无网格法作为一种新的计算方法,受到了广泛关注,已成功用于求解各种断裂力学问题。Sladek 等[106]采用无网格 Petrov-Galerkin(MLPG)法研究了磁电弹性材料裂纹问题。Li 等[107]采用 MLPG

与 FEM 耦合的方法处理了多种磁电弹性材料裂纹问题。Feng 等[108]采用 MLPG 法研究了磁电弹性材料裂纹的热冲击断裂问题。

随着磁电弹性材料断裂研究的不断深入,越来越多的问题将通过数值方法来求解。无网格法以及可能出现的新的计算方法也将随着更多磁电弹性材料断裂问题的解决而不断发展与完善。

2 结束语

全面综述了近年来磁电弹性材料断裂力学问题研究的现状。尽管磁电弹性材料断裂分析已取得了大量的研究成果,但仍然处于探索阶段,有许多工作有待进一步深入研究:

(1)磁电耦合场对裂纹扩展的实验研究。

(2)磁电弹性材料中自由电荷和电流对裂纹扩展的影响。

(3)磁电弹性材料的损伤疲劳研究以及和宏观裂纹的相互响应分析。

(4)无网格法以及新的数值方法在磁电弹性材料断裂问题中的进一步应用。

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(下转第 63 页)

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Free Axial Vibrations of a Non-uniform Rod

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Abstract: In this paper, the free axial vibrations of a rod with variable cross section are investigated. The area of the cross section of the rod varies exponentially along the axes of the rod. In terms of the proposed transformation, the governing equations of the free vibration are transformed as degenerated hyper-geometry equations, the solutions of which can be expressed by Kummer functions. The frequency equations and mode functions are obtained under three kinds of boundary conditions. Since the frequency equations are transcendental equations, the natural frequencies are solved with a numeric method. For some special cases, the explicit expressions of natural frequencies can be obtained.

Key words: rod with variable cross section; axial vibration; free vibrations; Kummer functions; hyper-geometry equation

(上接第 45 页)

Current Research on Fracture Mechanics of Magneto-electro-elastic Materials

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Abstract: Many mechanical researchers have fixed their attentions at the fracture of magneto-electro-elastic material recently. In this paper, the current research on the fracture mechanics of magneto-electro-elastic materials is reviewed. The main topics are composed of anti-plane, plane and penny-shaped crack problems as well as the application of finite element method, boundary element method and meshless method to the fracture of magneto-electro-elastic materials.

Key words: magneto-electro-elastic materials; fracture mechanics; meshless method